SAT Math Hard Practice Quiz

Numbers and Operations

1. A bag contains tomatoes that are either green or red. The ratio of green tomatoes to red tomatoes in the bag is 4 to 3. When five green tomatoes and five red tomatoes are removed, the ratio becomes 3 to 2. How many red tomatoes were originally in the bag?

(A) 12
(B) 15
(C) 18
(D) 24
(E) 30

2. If each digit in an integer is greater than the digit to the left, the integer is said to be "monotonic". For example, 12 is a monotonic integer since $2 > 1$. How many positive two-digit monotonic integers are there?

(A) 28
(B) 32
(C) 36
(D) 40
(E) 44

$$a,\ 2a-1,\ 3a-2,\ 4a-3,\ \ldots$$

3. For a particular number a, the first term in the sequence above is equal to a, and each term thereafter is 7 greater than the previous term. What is the value of the 16^{th} term in the sequence?

4. If p is a prime number, how many factors does p^3 have?

(A) One
(B) Two
(C) Three
(D) Four
(E) Five

5. How many integers between 10 and 500 begin and end in 3 ?

6. A particular integer N is divisible by two different prime numbers p and q. Which of the following must be true?

I. N is not a prime number.

II. N is divisible by pq.

III. N is an odd integer.

(A) I only
(B) II only
(C) I and II only
(D) I and III only
(E) I, II, and III

7. A perfect square is an integer that is the square of an integer. Suppose that m and n are positive integers such that $mn > 15$. If $15mn$ is a perfect square, what is the least possible value of mn ?

8. M is a set of six consecutive even integers. When the least three integers of set M are summed, the result is x. When the greatest three integers of set M are summed, the result is y. Which of the following is true?

(A) $y = x - 18$
(B) $y = x + 18$
(C) $y = 2x$
(D) $y = 2x + 4$
(E) $y = 2x + 6$

9. A three-digit number, XYZ, is formed of three different non-zero digits X, Y, and Z. A new number is formed by rearranging the same three digits. What is the greatest possible difference between the two numbers? (For example, 345 could be rearranged into 435, for a difference of $435 - 345 = 90$.)

10. An integer is subtracted from its square. The result could be which of the following?

(A) A negative integer.
(B) An odd integer.
(C) The product of two consecutive even integers.
(D) The product of two consecutive odd integers.
(E) The product of two consecutive integers.

SAT Math Hard Practice Quiz

Algebra and Functions

1. Let m be an even integer. How many possible values of m satisfy $\sqrt{m+7} \leq 3$?

(A) One
(B) Two
(C) Three
(D) Four
(E) Five

2. Let $\boxed{x}$ be defined by $\boxed{x} = \dfrac{x+3}{x-1}$ for any x such that $x \neq 1$. Which of the following is equivalent to $\boxed{x} - 1$?

(A) $\dfrac{x+2}{x-1}$

(B) $\dfrac{4}{x-1}$

(C) $\dfrac{2x+4}{x-1}$

(D) $\dfrac{2}{x-1}$

(E) $\dfrac{x+2}{x-2}$

3. Let a and b be numbers such that $a^3 = b^2$. Which of the following is equivalent to $b\sqrt{a}$?

(A) $b^{2/3}$

(B) $b^{4/3}$

(C) b^2

(D) b^3

(E) b^4

4. Let m and n be positive integers such that one-third of m is n less than one-half of m. Which of the following is a possible value of m?

(A) 15
(B) 21
(C) 24
(D) 26
(E) 28

5. If a and b are numbers such that $(a-4)(b+6) = 0$, then what is the smallest possible value of $a^2 + b^2$?

6. Let $f(x) = ax^2$ and $g(x) = bx^4$ for any value of x. If a and b are positive constants, for how many values of x is $f(x) = g(x)$?

(A) None
(B) One
(C) Two
(D) Three
(E) Four

7. Let a and b be numbers such that $30 < a < 40$ and $50 < b < 70$. Which of the following represents all possible values of $a - b$?

(A) $-40 < a - b < -20$
(B) $-40 < a - b < -10$
(C) $-30 < a - b < -20$
(D) $-20 < a - b < -10$
(E) $-20 < a - b < 30$

$$\frac{x}{3} + \frac{y}{12} = z$$

8. In the equation shown above, x, y, and z are positive integers. All of the following could be a possible value of y EXCEPT

(A) 4
(B) 6
(C) 8
(D) 12
(E) 20

$$\sqrt{72} + \sqrt{72} = m\sqrt{n}$$

9. In the equation above, m and n are integers such that $m > n$. Which of the following is the value of m?

(A) 6
(B) 12
(C) 16
(D) 24
(E) 48

t	0	1	2
$N(t)$	128	16	2

10. The table above shows some values for the function N. If $N(t) = k \cdot 2^{-at}$ for positive constants k and a, what is the value of a?

(A) -3

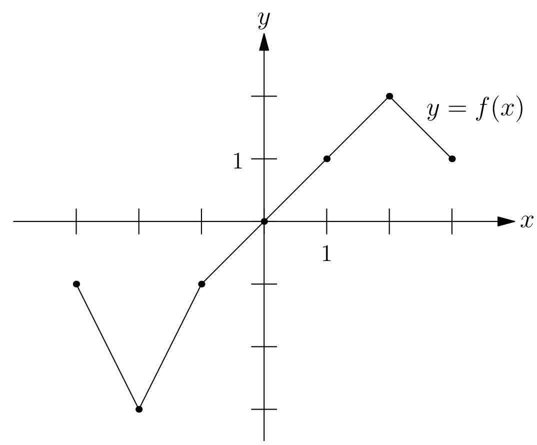
(B) -2

(C) $\frac{1}{3}$

(D) 2

(E) 3

11. Amy is two years older than Bill. The square of Amy's age in years is 36 greater than the square of Bill's age in years. What is the sum of Amy's age and Bill's age in years?

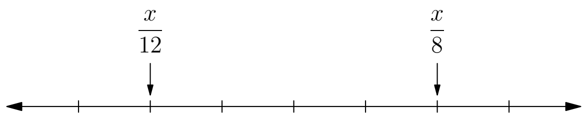


12. The function f is graphed in its entirety above. If the function g is defined so that $g(x) = f(-x)$, then for what value of x does g attain its maximum value?

(A) -3
(B) -2
(C) 0
(D) 2
(E) 3

13. If $(x+1)^2 = 4$ and $(x-1)^2 = 16$, what is the value of x?

(A) -3
(B) -1
(C) 1
(D) 3
(E) 5



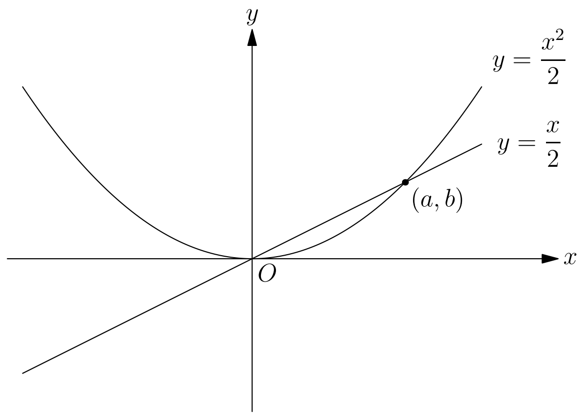
14. On the number line above, the tick marks correspond to consecutive integers. What is the value of x?

15. The value of y increased by 12 is directly proportional to the value of x decreased by 6. If $y = 2$ when $x = 8$, what is the value of x when $y = 16$?

(A) 8
(B) 10
(C) 16
(D) 20
(E) 28

16. Two cars are racing at a constant speed around a circular racetrack. Car A requires 15 seconds to travel once around the racetrack, and car B requires 25 seconds to travel once around the racetrack. If car A passes car B, how many seconds will elapse before car A once again passes car B?

Geometry



1. The curve $y = x^2/2$ and the line $y = x/2$ intersect at the origin and at the point (a, b), as shown in the figure above. What is the value of b?

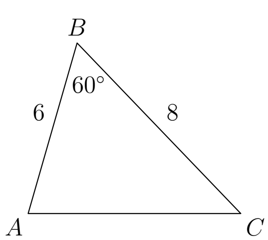
(A) $\frac{1}{8}$

(B) $\frac{1}{4}$

(C) $\frac{1}{2}$

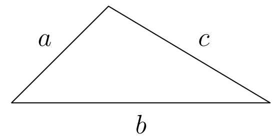
(D) 1

(E) 2



2. In the figure above, $AB = 6$ and $BC = 8$. What is the area of triangle ABC?

(A) $12\sqrt{2}$
(B) $12\sqrt{3}$
(C) $24\sqrt{2}$
(D) $24\sqrt{3}$
(E) $36\sqrt{3}$



Note: Figure not drawn to scale.

3. In the figure above, $3 < a < 5$ and $6 < b < 8$. Which of the following represents all possible values of c?

(A) $0 < c < 3$
(B) $1 < c < 3$
(C) $0 < c < 13$
(D) $1 < c < 13$
(E) $3 < c < 13$

4. Line l goes through points P and Q, whose coordinates are $(0, 1)$ and $(b, 0)$, respectively. For which of the following values of b is the slope of line l greater than $-\frac{1}{2}$?

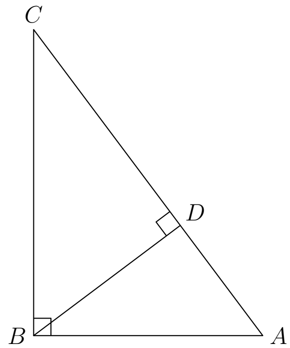
(A) $\frac{1}{2}$

(B) 1

(C) $\frac{3}{2}$

(D) $\frac{5}{3}$

(E) $\frac{5}{2}$



5. In the figure above, $AB = 6$ and $BC = 8$. What is the length of segment $\overline{BD}$?

(A) 2

(B) $\frac{12}{5}$

(C) 4

(D) $\frac{24}{5}$

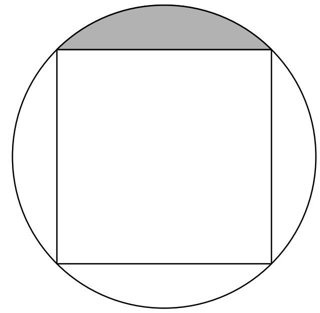
(E) 6

6. If four distinct lines lie in a plane, and exactly two of them are parallel, what is the least possible number of points of intersection of the lines?

(A) Two
(B) Three
(C) Four
(D) Five
(E) More than five

7. The perimeter of a particular equilateral triangle is numerically equal to the area of the triangle. What is the perimeter of the triangle?

(A) 3
(B) 4
(C) $4\sqrt{3}$
(D) $12\sqrt{3}$
(E) $18\sqrt{3}$



8. In the figure above, a square is inscribed in a circle. If the area of the square is 36, what is the perimeter of the shaded region?

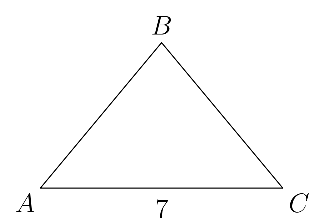
(A) $6 + \frac{3\sqrt{2}}{2}\pi$

(B) $6 + 3\pi$

(C) $6 + 3\sqrt{2}\pi$

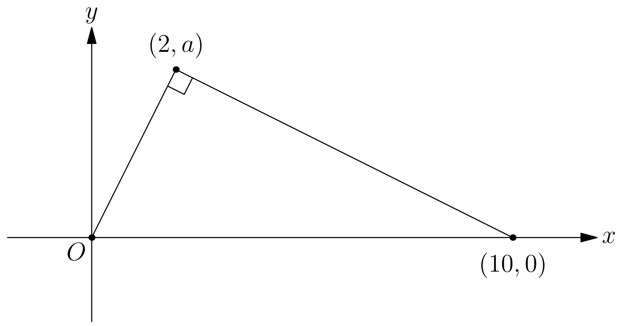
(D) $36 + 6\sqrt{2}\pi$

(E) $\frac{9}{2}\pi - 9$



Note: Figure not drawn to scale.

9. In the figure above, $AC = 7$ and $AB = BC$. What is the smallest possible integer value of AB?



10. In the figure above, two line segments in the x-y plane form a right triangle with the x-axis. What is the value of a?

(A) $2\sqrt{2}$
(B) 4
(C) 5
(D) $4\sqrt{2}$
(E) $5\sqrt{2}$

11. The perimeter of square $ABCD$ is x, and the perimeter of isosceles triangle EFG is y. If $AB = EF = FG$, which of the following must be true?

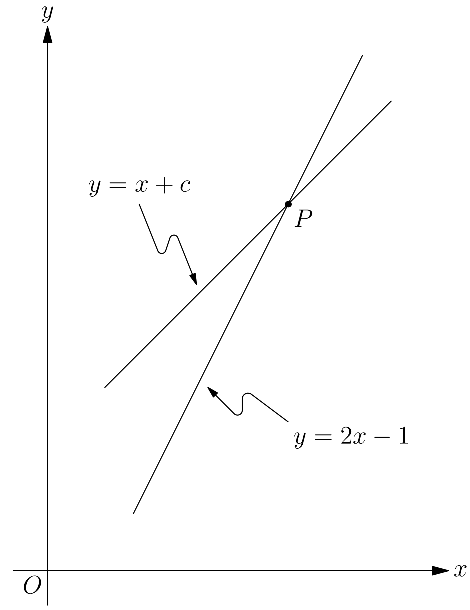
(A) $0 < y < \dfrac{x}{4}$

(B) $\dfrac{x}{4} < y < \dfrac{x}{2}$

(C) $\dfrac{x}{2} < y < x$

(D) $x < y < 2x$

(E) $2x < y < 4x$



Note: Figure not drawn to scale.

12. In the x-y plane, the lines $y = 2x - 1$ and $y = x + c$ intersect at point P, where c is a positive number. Portions of these lines are shown in the figure above. If the value of c is between 1 and 2, what is one possible value of the x-coordinate of P?

Data, Statistics, and Probability

1. The first term of a sequence is the number n, and each term thereafter is 5 greater than the term before. Which of the following is the average (arithmetic mean) of the first nine terms of this sequence?

(A) $n + 20$
(B) $n + 180$
(C) $2n$
(D) $2n + 40$
(E) $9n + 180$

2. The average (arithmetic mean) of a particular set of seven numbers is 12. When one of the numbers is replaced by the number 6, the average of the set increases to 15. What is the number that was replaced?

(A) -20
(B) -15
(C) -12
(D) 0
(E) 12

3. Let a, b, and c be positive integers. If the average (arithmetic mean) of a, b, and c is 100, which of the following is NOT a possible value of any of the integers?

(A) 1
(B) 100
(C) 297
(D) 298
(E) 299

4. M is a set consisting of a finite number of consecutive integers. If the median of the numbers in set M is equal to one of the numbers in set M, which of the following must be true?

I. The average (arithmetic mean) of the numbers in set M equals the median.

II. The number of numbers in set M is odd.

III. The sum of the smallest number and the largest number in set M is even.

(A) I only
(B) II only
(C) I and II only
(D) I and III only
(E) I, II, and III

Numbers and Operations

1. B (Estimated Difficulty Level: 5)

The number of green and red tomatoes are $4n$ and $3n$, respectively, for some integer n. In this way, we can be sure that the green-to-red ratio is $4n/3n = 4/3$. We need to solve the equation:

$$\frac{4n-5}{3n-5} = \frac{3}{2}.$$

Cross-multiplying, $8n - 10 = 9n - 15$ so that $n = 5$. There were $3n$, or 15, red tomatoes in the bag.

Working with the answers may be easier. If answer A is correct, then there were 16 green tomatoes and 12 red tomatoes, in order to have the 4 to 3 ratio. But removing five of each gives 11 green and 7 red, which is not in the ratio of 3 to 2. If answer B is correct, then there were 20 green tomatoes and 15 red tomatoes, since $20/15 = 4/3$. Removing five of each gives 15 green and 10 red, and $15/10 = 3/2$, so answer B is correct.

2. C (Estimated Difficulty Level: 4)

From 10 to 19, 12 and up (eight numbers) are monotonic. Among the numbers from 20 to 29, seven (23 and up) are monotonic. If you can see a pattern in counting problems like this, you can save a lot of time. Here, the 30s will have 6 monotonic numbers, the 40s will have 5, and so forth. You should find $8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 + 0 = 36$ total monotonic numbers.

3. 113 (Estimated Difficulty Level: 5)

Since the second term is 7 greater than the first term, $(2a - 1) - a = 7$ so that $a = 8$. The sequence is 8, 15, 22, ... You can either continue to write out the sequence until the 16^{th} term, or realize that the 16^{th} term is $16a - 15 = 16(8) - 15 = 128 - 15 = 113$.

4. D (Estimated Difficulty Level: 4)

The answer must be true for any value of p, so plug in an easy (prime) number for p, such as 2. The factors of $2^3 = 8$ are 1, 2, 4, and 8, so answer D is correct.

In general, since p is prime, the only numbers that go into p^3 without a remainder are 1, p, p^2, and p^3.

5. 11 (Estimated Difficulty Level: 4)

For the two-digit numbers, only 33 begins and ends in 3. For three-digit numbers, the only possibilities are: 303, 313, ..., 383, and 393. We found ten three-digit numbers, and one two-digit number, for a total of 11 numbers that begin and end in 3.

Yes, this was a counting problem soon after another counting problem. But this one wasn't so bad, was it?

6. C (Estimated Difficulty Level: 4)

This type of SAT math question contains three separate mini-problems. (This kind of question is also known as "one of those annoying, long, SAT math questions with roman numerals"). Let's do each mini-problem in order.

First, recall that a prime number is only divisible by itself and 1, and that 1 is not a prime number. So, statement I must be true, since a number that can be divided by two prime numbers can't itself be prime.

Next, recall that every number can be written as a product of a particular bunch of prime numbers. Let's say that N is divisible by 3 and 5. Then, N is equal to $3 \cdot 5 \cdot p_1 \cdot p_2 \cdots$, where p_1, p_2, etc. are some other primes. So, N is divisible by $3 \cdot 5 = 15$. Statement II must be true.

Finally, remember that 2 is a prime number. So, N could be 6, since $6 = 2 \cdot 3$. Statement III isn't always true, making C the correct answer.

7. 60 (Estimated Difficulty Level: 5)

First, note that $15mn = 3 \cdot 5 \cdot mn$. We need $\sqrt{3 \cdot 5 \cdot mn}$ to be an integer. We could have, for example, $m = 3$ and $n = 5$ since $\sqrt{3 \cdot 5 \cdot 3 \cdot 5} = \sqrt{3^2 \cdot 5^2} = 15$, except that the problem requires that $mn > 15$. (This is a hard problem for a reason, after all!) If $m = 3 \cdot 2$ and $n = 5 \cdot 2$ then $15mn = (3 \cdot 5)(3 \cdot 2)(5 \cdot 2) = 3^2 \cdot 5^2 \cdot 2^2$. That way, $\sqrt{15mn} = \sqrt{3^2 \cdot 5^2 \cdot 2^2} = 30$ is still an integer, making the least possible value of mn equal to $6 \cdot 10 = 60$.

8. B (Estimated Difficulty Level: 4)

A good opportunity to plug in real numbers! For example, suppose set M consists of the integers: 2, 4, 6, 8, 10, and 12. The sum of the least three is 12 and the sum of the greatest three is 30, so answer B is correct.

You say you want an algebraic solution? Suppose that n is the first even integer. The remaining integers are then $n+2$, $n+4$, $n+6$, $n+8$, and $n+10$. The sum of the least three of these integers is $x = n+(n+2)+(n+4) = 3n+6$, and the sum of the greatest three of these integers is $y = (n+6)+(n+8)+(n+10) = 3n+24$. So, $y-x = 18$, or $y = x + 18$.

9. 792 (Estimated Difficulty Level: 5)

To get the greatest difference, we want to subtract a small number from a large one, so we will need the digit 9 and the digit 1, in order to make a number in the 100's and a number in the 900's. The large number will look like $9N1$ and the small number will look like $1N9$, where N is a digit from 2 to 8. You will find that, no matter what you make N, the difference is 792.

10. E (Estimated Difficulty Level: 5)

Suppose that the integer is n. The result of subtracting n from its square is $n^2 - n = n(n-1)$, which is the product of two consecutive integers, so answer E is correct.

Notice that if you multiply any two consecutive integers, the result is always even, since it is the product of an even integer and an odd integer. To win an Erik The Red Viking Hat, see if you can determine why the result is never a negative integer.

Algebra and Functions

1. E (Estimated Difficulty Level: 5)

The answers suggest that there aren't that many possibilities. So, make up some even integers, plug them in for m, and see if they work! Since we can't take the square root of a negative number, m can't be less than -6. Also, if $m = 2$, then $\sqrt{m+7} = 3$, but any larger value of m won't work. So, the possible values are -6, -4, -2, 0, and 2. (Don't forget that zero is a perfectly good even integer.)

2. B (Estimated Difficulty Level: 4)

Plug in real numbers for x! You can plug in anything other than 1. If you set $x = 2$, then $\boxed{x} - 1 = 5 - 1 = 4$. Now go through the answers, plugging in 2 for x. You will find that answers A and B are both equal in value to 4. If this happens, simply plug in another number. (You don't need to retry the answers that were wrong.) If $x = 0$, then $\boxed{x} - 1 = -3 - 1 = -4$, and only answer B is also -4, so that is the correct answer.

If you love algebra, here is how to do it:

$$\begin{aligned}\boxed{x} - 1 &= \frac{x+3}{x-1} - 1\\ &= \frac{x+3}{x-1} - \frac{x-1}{x-1}\\ &= \frac{x+3-(x-1)}{x-1}\\ &= \frac{4}{x-1}\end{aligned}$$

Be still my beating heart!

3. B (Estimated Difficulty Level: 5)

Solve the first equation given for a: $a = b^{2/3}$. Then $\sqrt{a} = a^{1/2} = b^{1/3}$. (You really need to know your exponent rules for this one.) So, $b\sqrt{a} = b \cdot b^{1/3} = b^{4/3}$. Using real numbers also works here, but it may be hard to come up with two that work for a and b (such as $a = 4$ and $b = 8$).

4. C (Estimated Difficulty Level: 4)

Translate the words into an algebraic equation:

$$\frac{m}{3} = \frac{m}{2} - n.$$

Multiplying both sides by 6 (the common denominator) gives $2m = 3m - 6n$, or $m = 6n$. So, m must be a positive multiple of 6, which means that answer C is correct.

5. 16 (Estimated Difficulty Level: 4)

Since $(a-4)(b+6) = 0$, the possible solutions are: $a = 4$ and b is anything, or $b = -6$ and a is anything. Now, the expression $a^2 + b^2$ is made smallest by choosing a and b to be close to zero as possible. So, $a = 4$ and $b = 0$ will give us the smallest value of $a^2 + b^2$, namely, 16. Using the other solution would give $a^2 + 36$, which will always be bigger than 16.

6. D (Estimated Difficulty Level: 5)

This is a tough one. For $f(x)$ to be equal to $g(x)$ for all x, we need $ax^2 = bx^4$. First, notice that if $x = 0$, both sides are zero, so $x = 0$ is a solution. If x is not zero, we can divide both sides of the equation by x^2 to get: $a = bx^2$. Solving for x results in $x = \pm\sqrt{a/b}$. This makes three solutions total, so answer D is correct. It may help to plug in numbers for a and b to make this problem more concrete.

7. B (Estimated Difficulty Level: 5)

To make $a - b$ as large as possible, we need to make a as large as possible and b as small as possible. So, $a - b$ has to be less than $40 - 50 = -10$. To make $a - b$ as small as possible, we need to make a as small as possible and b as large as possible. So, $a - b$ has to be greater than $30 - 70 = -40$. The expression that gives all possible values of $a - b$ is then $-40 < a - b < -10$.

8. B (Estimated Difficulty Level: 5)

Solve the equation for y. You should get: $y = 12z - 4x$. Factor out a 4 from the right-hand side: $y = 4(3z - x)$. Since z and x are integers, $3z - x$ is an integer, so that y is a multiple of 4. Only answer B is not a multiple of 4, so it can't be a possible value of y.

You could also combine the fractions on the left-hand side of the equation to get:

$$\frac{4x + y}{12} = z.$$

For z to be an integer, $4x + y$ must be a multiple of 12. Try plugging in various integers for x and y to get multiples of 12; you should find that y can take on all of the values in the answers except for 6.

9. B (Estimated Difficulty Level: 4)

Combining the two terms on the left-hand side of the equation gives us $2\sqrt{72}$, but that doesn't give us what we need, which is $m > n$.

For the SAT test, you should know how to rewrite and simplify radicals. Here, $\sqrt{72} = \sqrt{36 \cdot 2} = 6\sqrt{2}$, so the left-hand side is equal to $12\sqrt{2}$, making $m = 12$ and $n = 2$.

10. E (Estimated Difficulty Level: 5)

Hint: if you see a zero in a table problem like this one, try to use it first! When you plug in 0 for t, you get $N(0) = k \cdot 2^{-0} = k \cdot 1 = k$, which means that $k = 128$.

Next, try plugging in 1 for t: $N(1) = 128 \cdot 2^{-a}$. From the table, $N(1) = 16$, so that $128 \cdot 2^{-a} = 16$, or $2^{-a} = 1/2^a = 1/8$. Since $8 = 2^3$, $a = 3$. (Your calculator may also help here, but try to understand how to do it without it.)

11. 18 (Estimated Difficulty Level: 4)

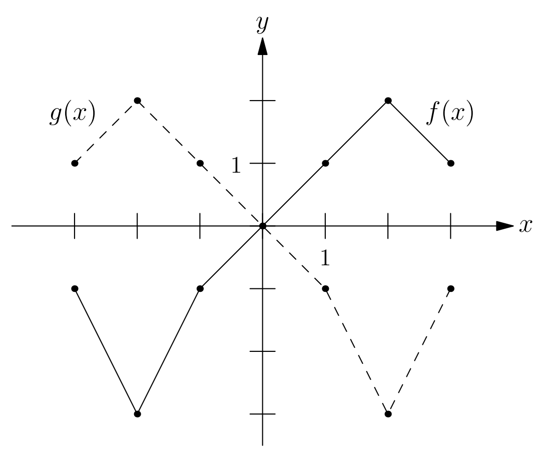
Let a be Amy's age and b be Bill's age. The problem tells us that: $a = b + 2$ and $a^2 = b^2 + 36$. One way to do this is to plow ahead, substitute for one variable and solve for the other (a bit messy!). But SAT questions are designed to be solved without tedious calculations and/or messy algebra. Let's try doing the problem using the SAT way, not the math teacher way.

First, notice that the second equation can be written as: $a^2 - b^2 = 36$. This is a difference of two squares, and is the same as: $(a+b)(a-b) = 36$. The first equation can be written as: $a - b = 2$. This means that the second equation is just $(a + b) \cdot 2 = 36$, so that $a + b = 18$. We don't know what a and b are, and we don't even care!

12. B (Estimated Difficulty Level: 4)

You can obtain the graph of $y = f(-x)$ by "flipping" the graph of $y = f(x)$ across the y-axis. For example, if the point $(3, 1)$ is on the graph of $f(x)$, then the point $(-3, 1)$ must be on the graph of $f(-x)$, since $f(-(-3)) = f(3) = 1$.

The figure below tells the story. Here, the function $g(x) = f(-x)$ is shown as a dashed line:



From the graph, $g(x)$ is maximum when $x = -2$.

13. A (Estimated Difficulty Level: 4)

A good "skip-the-algebra" way to do this problem is to use the answers by plugging them into x until the two given equations work. Using answer A, you should find that $(-3+1)^2 = (-2)^2 = 4$ and $(-3-1)^2 = (-4)^2 = 16$, so answer A is correct.

You must have the algebraic solution, you say? Try taking the square root of both sides of the equations, but don't forget that there are two possible solutions when you do this. The first equation gives: $x + 1 = \pm 2$ so that $x = 1$ or $x = -3$. The second equation gives $x - 1 = \pm 4$ so that $x = 5$ or $x = -3$. The only solution that works for both equations is $x = -3$.

14. 96 (Estimated Difficulty Level: 5)

Since the tick marks correspond to consecutive integers, and it takes four "steps" to go from $x/12$ to $x/8$, we know that $x/8$ is four greater than $x/12$. (Or, think of the spaces between the tick marks: there are four spaces and each space is length 1, so the distance from $x/12$ to $x/8$ is 4.) In equation form:

$$\frac{x}{8} = \frac{x}{12} + 4.$$

Multiplying both sides by 24 gives: $3x = 2x + 4 \cdot 24$ so that $x = 96$.

15. B (Estimated Difficulty Level: 5)

First, recall that if y is proportional to x, then $y = kx$ for some constant k. So, "y increased by 12 is directly proportional to x decreased by 6" translates into the math equation: $y + 12 = k(x - 6)$. Plugging in $y = 2$ and $x = 8$ gives $14 = k \cdot 2$ so that $k = 7$. Our equation is now: $y + 12 = 7(x - 6)$. Plugging in 16 for y gives $28 = 7(x - 6)$ so that $x - 6 = 4$, or $x = 10$.

16. 75/2 or 37.5 (Estimated Difficulty Level: 5)

To make this problem more concrete, make up a number for the circumference of the racetrack. It doesn't really matter what number you use; I'll use 75 feet. Since speed is distance divided by time, the speed of car A is $75/15 = 5$ feet per second, and the speed of car B is $75/25 = 3$ feet per second. (I picked 75 mostly because it is divided evenly by 15 and 25.) Every second, car A gains 2 feet on car B. To pass car B, car A must gain 75 feet on car B. This will require $75/2 = 37.5$ seconds.

You may be thinking, "Whoa, tricky solution!" Here is the mostly straightforward but somewhat tedious algebraic solution. Once again, I'll use 75 feet for the circumference of the track. Suppose that you count time from when car A first passes car B. Then, car A travels a distance $(75/15)t = 5t$ feet after t seconds. (Remember that distance = speed $\times$ time.) For example, after 15 seconds, car A has traveled a distance $5 \cdot 15 = 75$ feet, and after 30 seconds, car A has traveled a distance $5 \cdot 30 = 150$ feet. Similarly, car B travels a distance $(75/25)t = 3t$ feet after t seconds. When the two cars pass again, car A has traveled 75 feet more than car B: $5t = 3t + 75$. Solving for t gives: $2t = 75$, or $t = 75/2 = 37.5$ seconds.

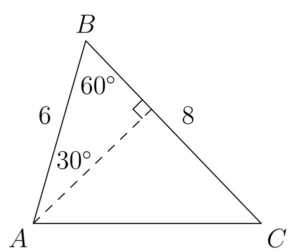
Geometry

1. C (Estimated Difficulty Level: 4)

To determine where two curves intersect, set the equations equal to one another and solve for x. (Hint: know this for the SAT!) In this question, we need to figure out which values of x satisfy: $x^2/2 = x/2$. If $x = 0$, this equation works, but we need the solution when $x \neq 0$. Dividing both sides of the equation by x gives: $x/2 = 1/2$ so that $x = 1$. By plugging in $x = 1$ to either of the two curves, you should find that $y = 1/2$. So, the point of intersection is $(1, 1/2)$, making answer C the correct one.

2. B (Estimated Difficulty Level: 5)

This is the kind of problem that would be too hard and/or require things you aren't expected to know for the SAT (such as trigonometry), *unless* you draw a construction line in the figure. (This is a very hard question anyway.) In this case, you want to draw a line from A perpendicular to the opposite side of the triangle:



This forms a 30-60-90 triangle whose hypotenuse has length 6. Now, use the 30-60-90 triangle diagram given to you at the beginning of each SAT math section: The length of the side opposite the 30° angle is 3, and the length of the side opposite the 60° angle (the dashed line) is $3\sqrt{3}$.

So finally, if the base of the triangle is segment $\overline{BC}$, then the dashed line is the height of the triangle, and the area of the triangle is $(1/2) \cdot 8 \cdot 3\sqrt{3} = 12\sqrt{3}$.

3. D (Estimated Difficulty Level: 5)

You need to know the "third-side rule" for triangles to solve this question: *The length of the third side of a triangle is less than the sum of the lengths of the other two sides and greater than the positive difference of the lengths of the other two sides.* Applied to this question, the first part of the rule says that the value of c must be less than $a + b$. Since we are interested in *all* possible values of c, we need to know the greatest possible value of $a + b$. With $a < 5$ and $b < 8$, $a + b < 13$ so that c must be less than 13.

For the second part of the rule, c must be greater than $b - a$. (Note that b is always bigger than a, so that $b - a$ is positive.) We are interested in *all* possible values of c, so we need to know the least possible value of $b - a$. The least value occurs when b is as small as possible and a is as large as possible: $b - a > 6 - 5 = 1$. Then, c must be greater than 1. Putting this together, $1 < c < 13$, making answer D the correct one.

4. E (Estimated Difficulty Level: 4)

First, calculate the slope of line l using the given points:

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{0 - 1}{b - 0} = -\frac{1}{b}.$$

At this point, a good approach is to work with the answers by plugging them into the expression for slope above until you get a value greater than $-1/2$. For example, using answer A gives a slope of $-1/(1/2) = -2$, which is not greater than $-1/2$, so answer A is incorrect. You should find that answer E is the correct one, since $-1/(5/2) = -2/5$ is greater than $-1/2$. (Knowing the decimal equivalents of basic fractions will really help speed this process up.)

Here is the algebraic solution:

$$-\frac{1}{b} > -\frac{1}{2} \quad \Rightarrow \quad \frac{1}{b} < \frac{1}{2} \quad \Rightarrow \quad b > 2.$$

(Remember to flip the inequality when multiplying by negative numbers or when taking the reciprocal of both sides.) Only answer E makes $b > 2$.

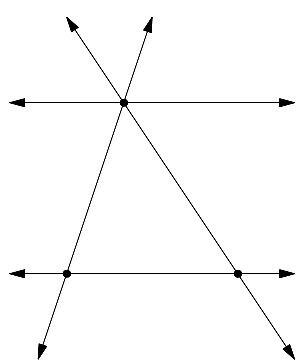
5. D (Estimated Difficulty Level: 5)

Since ABC is a right triangle, the length of segment $\overline{AC}$ is $\sqrt{6^2 + 8^2} = 10$. (Hint: you will see 3-4-5 and 6-8-10 triangles a lot on the SAT.) The area of triangle ABC is $(1/2) \cdot b \cdot h$, where b is the base and h is the height of triangle ABC.

The key thing to remember for this problem is that the base can be *any* of the three sides of a triangle, not just the side at the bottom of the diagram. If the base is AB, then the height is BC and the area of the triangle is $(1/2)(6)(8) = 24$. If the base is AC, then the height is BD and the area of the triangle is still 24. This means that $(1/2)(AC)(BD) = (1/2)(10)(BD) = 24$ so that $BD = 24/5$.

6. B (Estimated Difficulty Level: 4)

Draw a diagram for this problem! With exactly two parallel lines, the other two lines cannot be parallel to themselves or to the first two lines. Your diagram may seem to suggest five points of intersection; however, the point of intersection of the two non-parallel lines can overlap with a point of intersection on one of the parallel lines:



From the figure, the least possible number of intersection points is then three.

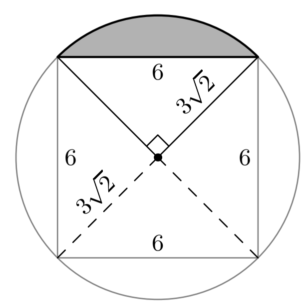
7. D (Estimated Difficulty Level: 5)

One formula that a good math student such as yourself may want to memorize for the SAT is the area of an equilateral triangle. If the length of each side of the triangle is s, then the area is $\sqrt{3}s^2/4$. The perimeter of this triangle is $3s$.

Now, since the perimeter equals the area for this triangle, we have: $3s = \sqrt{3}s^2/4$ so that $3 = \sqrt{3}s/4$ and $s = 12/\sqrt{3} = 4\sqrt{3}$. The perimeter is then $12\sqrt{3}$, making answer D the correct one. (Did you get $s = 4\sqrt{3}$ and then choose answer C? Sorry about that.)

8. A (Estimated Difficulty Level: 5)

For many difficult SAT questions, it can be very helpful to know some "extra" math along with the "required" math. First, when a square is inscribed in a circle, the diagonals are diameters of the circle. Second, the diagonals of a square meet at right angles. Third, a diagonal of a square is $\sqrt{2}$ times as long as the length of one of the sides. (A diagonal of a square makes a 45-45-90 triangle with two sides.) For this question, the length of each side of the square is 6 (since the area is $6^2 = 36$), and the length of a diagonal is $6\sqrt{2}$, so the radius of the circle is $3\sqrt{2}$, as shown below:



A final piece of needed math: the arc length of a portion of a circle is the circumference times the central angle of the arc divided by 360°. Here, the central angle is 90°, so the needed arc length (shown darkened in the figure above) is just 1/4 times the circle's circumference. The arc length is then $2\pi r/4 = 2\pi \cdot 3\sqrt{2}/4 = 3\pi\sqrt{2}/2$ and the perimeter of the shaded region is $6 + 3\pi\sqrt{2}/2$.

9. 4 (Estimated Difficulty Level: 5)

You need to know half of the "third-side rule" for triangles to solve this question: *The length of the third side of a triangle is less than the sum of the lengths of the other two sides.* For this question, we will make AC the third side.

Now, suppose that the length of each of the other two sides of the triangle is x, so that $AB = BC = x$. Then, the third-side rule says that AC is less than the sum of AB and BC: $7 < x + x$. Simplifying gives: $2x > 7$ so that $x > 3.5$. The smallest possible integer value for x is 4.

10. B (Estimated Difficulty Level: 5)

One way to do this question is to use the fact that the product of the slopes of two perpendicular lines (or line segments) is -1. The slope of the line segment on the left is $(a - 0)/(2 - 0) = a/2$. The slope of the line segment on the right is $(0 - a)/(10 - 2) = -a/8$. The two slopes multiply to give -1:

$$\frac{a}{2} \cdot \frac{-a}{8} = -\frac{a^2}{16} = -1.$$

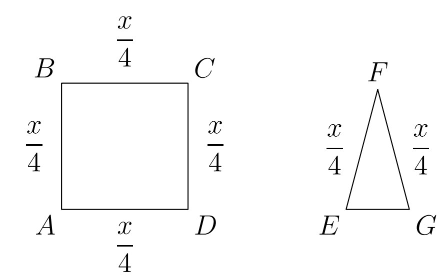
Solving for a gives $a^2 = 16$ so that $a = 4$. A messier way to do this problem is to use the distance formula and the Pythagorean theorem. The length of the line segment on the left is $\sqrt{2^2 + a^2}$, and the length of line segment on the right is $\sqrt{(10-2)^2 + (0-a)^2}$. Then, the Pythagorean theorem says that:

$$\left(\sqrt{2^2 + a^2}\right)^2 + \left(\sqrt{(10-2)^2 + (0-a)^2}\right)^2 = 10^2.$$

Simplifying the left-hand side gives: $2a^2 + 68 = 100$ so that $2a^2 = 32$. Then, $a^2 = 16$, making $a = 4$.

11. C (Estimated Difficulty Level: 5)

Make a diagram, and fill it in with the information that is given. (You should do this for any difficult geometry question without a figure.) Since the perimeter of square $ABCD$ is x, each side of the square has length $x/4$, so your figure should look something like this:



Now, use the third-side rule for triangles: *The length of the third side of a triangle is less than the sum of the lengths of the other two sides and greater than the positive difference of the lengths of the other two sides.* When the rule is applied to EG as the third side, we get: $0 < EG < x/2$. If y is the perimeter of the triangle, then $y = x/4 + x/4 + EG = x/2 + EG$. Solving for EG gives $EG = y - x/2$. Substituting into the inequality gives $0 < y - x/2 < x/2$ so that $x/2 < y < x$, making answer C the correct one. To make this problem less abstract, it may help to make up a number for the perimeter of the square. (A good choice might be 4 so that $x = 1$. You'll find $1/2 < y < 1$, the same as answer C when $x = 1$.)

12. $2 < x < 3$ (Estimated Difficulty Level: 5)

In order to determine at what point two lines intersect, set the equations of the lines equal to one another. In this case, we have: $2x - 1 = x + c$ so that $x = c + 1$. In other words, $x = c + 1$ is the x-coordinate of P, the point where the lines intersect. Now, if c is between 1 and 2, then $c + 1$ is between 2 and 3. Any value for the x-coordinate of P between 2 and 3 is correct.

SAT Math Hard Practice Quiz Answers

Data, Statistics, and Probability

1. A (Estimated Difficulty Level: 4)

The first nine terms of the sequence are:

$$n,\ n+5,\ n+10,\ n+15,\ \ldots,\ n+40.$$

(You should probably write all nine terms out to avoid mistakes.) Adding these terms up gives: $9n+180$. The average is the sum $(9n+180)$ divided by the number of terms (9). The average is then: $(9n+180)/9 = n+20$.

2. B (Estimated Difficulty Level: 5)

The average of a set of numbers is the sum of the numbers divided by the number of numbers:

$$\text{average} = \frac{\text{sum}}{N}.$$

We can solve this equation for the sum:

$$\text{sum} = \text{average} \times N.$$

Here, since there are 7 numbers and the average is 12, the sum of the numbers is $7 \times 12 = 84$. The sum of the new set of numbers is $7 \times 15 = 105$. Now, suppose that the seven numbers are a, b, c, d, e, f, and g, and that g gets replaced with the number 6. Then, we have:

$$a+b+c+d+e+f+g = 84,$$

and

$$a+b+c+d+e+f+6 = 105.$$

The second equation says that $a+b+c+d+e+f = 99$. Substituting into the first equation gives $99+g = 84$ so that $g = -15$.

3. E (Estimated Difficulty Level: 4)

Using the definition of average gives:

$$\frac{a+b+c}{3} = 100$$

so that $a+b+c = 300$. Since a, b, and c are all positive, the smallest possible value for any of the numbers is 1. The largest possible value of one of the three numbers then occurs when the other two numbers are both 1. In this case, the numbers are 1, 1, and 298, so that the largest possible value is 298. Answer E can not be a possible value, so it is the correct answer.

4. E (Estimated Difficulty Level: 5)

Plug in real numbers for set M to make this problem concrete. For example, if M is the set of consecutive integers from 1 to 5, then the median and average are both 3. If M is the set of consecutive integers from 1 to 4, then the median and average are both 2.5. From these examples, we can see that the number of numbers in set M needs to odd, otherwise the median is not an integer. Choice II must be true.

Also, if the number of numbers in a set of consecutive integers is odd, then when the first number is odd, the last number is odd. Or, when the first number is even, the last number is even. This is because the difference of the largest number and the smallest number will be even when the number of numbers is odd. Choice III must then be true, since the sum of two odd numbers or two even numbers is an even number.

At this point, the only answer with choices II and III is answer E, so that must be the correct answer. Why is choice I also correct? The average of a set of consecutive integers is equal to the average of the first and the last integers in the set. The average of two integers that are both odd or both even is the integer halfway between the two, which is also the median of the set. Whew!